THE MASS DEFECT OF BARYON STARS

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The gravitational mass defect $\Delta M = nm_n - M$ of neutron configurations has been investigated. Here M is the mass of the star, n is the number of neutrons it contains, and m_n is the neutron mass. It is shown that in the case of central densities $\rho(0)$ exceeding nuclear density by an order of magnitude ΔM exhibits anomalous behavior (according to Einstein's gravitational theory). The anomaly consists in that with increase in $\rho(0)$ the mass defect decreases and, in the case of central baryon densities exceeding 10^{40} cm^{-3}, becomes negative. This phenomenon is caused by the severe disruption of additivity of internal energy in an intense gravity field, where nonlinear effects are extremely important. In Newton's theory of gravitation ΔM is positive for all densities. Baryon stars with a negative mass defect have a colossal reserve of internal energy. This energy is of the order of the proper energy of the star itself.

1. The existence of baryon configurations with an anomalous (negative) mass defect was predicted in [1]. The essence of the anomaly is that the mass of a superdense body is greater than the sum of the rest masses of the baryons it contains. In the study mentioned we emphasized the importance of such stellar configurations for astrophysics, but no detailed investigation was made. The same subject was later discussed by Zel'dovich [2]. He demonstrated that $M < f nm_n$, where M is the observed mass of the star, n is the number of neutrons in it, m_n is the neutron mass, $f \approx 1.5\sqrt{g_{rr}(R)}$, R is the star radius, and g_{rr} is a component of the metric tensor. This estimate of the upper limit of mass M does not exclude the possibility of the existence of equilibrium configurations with mass $M > nm_n$. This article offers a more detailed analysis of the problem.

In the case of spherical baryon stars the gravitational mass defect is

$$\delta M = \frac{4\pi}{c^2}\int_0^R \left(\sqrt{g_{rr}(r)}-1\right)\rho(r)\ r^2\,dr, \tag{1.1}$$

where $\rho(r)$ is the proper energy density, R is the coordinate radius of the star, $g_{rr}(r)$ is the radial component of the metric tensor for the Schwarzschild internal solution. In essence, (1.1) represents the difference of the masses of the configuration with and without taking into account the gravitational interaction (characteristic mass); therefore, in all cases $\delta M > 0$ (everywhere $g_{rr} > 1$). The packing factor $\delta M/M$, where M is the observed star mass, was computed in [1]. It was found to attain fairly large values (see Table 1 in [1]). In the case of configurations consisting of an ideal baryon gas, the packing factor varies from several percent to 20% for the densest configuration, and in the case of models with a specific variant of a real gas it may even reach 55%.

In this article we shall not be concerned with the mass defect (1.1), but with some other value defined by the relation

$$\Delta M = mn - M, \tag{1.2}$$

where n is the number of baryons in the star, m is the sum of the rest masses of a proton and an electron, and M is the observed star mass. Henceforth expression (1.2) will be called the absolute gravitational mass defect. It is this value that is of particular interest to astrophysicists. Note that, in contrast to (1.2), the value of the mass defect (1.1) is to some degree dependent on the selection of the frame of reference and therefore is not an invariant characteristic of the star. Actually, (1.1) does not change except in transformations of the type $x'^{\alpha} = f^{\alpha}(x^{\beta})$, $(\alpha,\ \beta = 1,\ 2,\ 3)$ or $x'^{0} = f^{0}(x^{0})$, whereas (1.2) is invariant relative to any transformations of the coordinates and time.

Obviously, for ordinary celestial bodies in all cases $\Delta M > 0$. The computations of baryon configurations have demonstrated [1, 4, 11] that when central densities rise above a certain value $\rho_1(0)$ the absolute mass defect changes sign — it becomes negative. The value of $\rho_1(0)$ depends on the form of the equation of state for the baryon gas used in the computations. In models with a real baryon gas $\rho_1(0)$ has a lesser value than in models with an ideal gas. This can be attributed to the fact that in the case of a real gas, at densities greater than nuclear, an important role is played by the nuclear forces of repulsion between baryons. Naturally, the forces of repulsion facilitate the appearance of the considered effect. It follows that the values (1.1) and (1.2) of gravitational mass defects for models with a real gas are not entirely correct, because in this case they are determined not only by gravitation but also by the effect of nuclear forces (attraction and repulsion). In order to exclude the influence of nuclear forces on the absolute mass defect and investigate the phenomenon in purer form, in what follows we will be concerned only with models of superdense stars consisting of an ideal baryon gas. Then (1. 2) actually will represent the absolute gravitational mass defect.

2. In [1, 4, 11] the object was not to compute the mass defect. This was determined incidentally after computation of the masses of the configurations and the number of baryons in them. The number of baryons was computed graphically correct to the third decimal place. For this reason the accuracy of the values ΔM was naturally not greater than two decimal places. Special computations of the mass defect have again been made in order to obtain more precise results. The presence of hyperons and interaction between baryons were disregarded, in order to avoid complications unrelated to the considered problem. The computations of neutron configurations were made on electronic computers at the Joint Computation Center of the Academy of Sciences of the Armenian SSR and Erevan State University.

When we use Einstein's gravitational theory as a point of departure, configurations consisting of an ideal neutron gas are defined by the following system of equations [3, 10]:

$$\frac{du}{dr} = r^2 (\operatorname{sh} t - t),$$

$$\frac{dt}{dr} = -\frac{4\left(\operatorname{sh} t - 2\operatorname{sh}\frac{t}{2}\right)}{r\left(r - 2u\right)\left(\operatorname{ch} t - 4\operatorname{ch}\frac{t}{2} + 3\right)}\left[u + \frac{r^3}{3}\left(\operatorname{sh} t - 8\operatorname{sh}\frac{t}{2} + 3t\right)\right]. \quad (2.1)$$

$$n = A\int_0^R \left(1 - \frac{2u}{r}\right)^{-1/2} \operatorname{sh}^3\frac{t}{4}\, r^2\, dr.$$

Here r is a radial coordinate, u(r) is the "mass" in a sphere with radius r, n is the number of neutrons in the star, R is the coordinate radius of the star (determined from the condition t(r) = 0), u(R) = M is the star mass, $A = m_n^3 c^3/3\pi^2 h^3)$ is a constant, m_n is the neutron mass, and, finally,

$$t = 4\operatorname{arsh}(p_n/m_n c), \quad (2.2)$$

where $p_n = (3\pi^2)^{1/3} h N^{1/3}$ is the limiting momentum of neutrons, and N(r) is the neutron density. In equations (2. 1) a system of units is used in which the speed of light and the gravitational constant are equal to unity c = k = 1 and $K_n = m_n^4 c^5/$ $/(32\pi^2 h^3) = 1/4\pi$. In these units $A = 1.174 \times 10^{59}$.

As the initial conditions it is necessary to assign values of the functions u(r) and t(r) at the center of the configurations. We have

$$u(0) = 0; \quad t(0) \neq 0. \quad (2.3)$$

In this case each specific value of the parameter t(0) will correspond to a particular neutron configuration.

Neutron configurations were also computed for the case when Newton's gravitational law is used as the point of departure. In this case we knowingly admitted a certain inconsistency, extending our computations to include configurations consisting of a relativistic neutron gas. However, in this case we had a definite purpose in mind: comparison of the exact and approximate computations in order to clarify the role of relativism of the baryon gas and the curvature of space and thereby determine the cause of the anomaly in the absolute gravitational mass defect. When Newton's gravitational law is used, the parameters of the neutron configurations are determined from the equations

$$\frac{du}{dr} = r^2 (\operatorname{sh} t - t),$$

$$\frac{dt}{dr} = -\frac{3u\,(\operatorname{sh} t - t)}{r^2\,[\operatorname{ch} t - 4\operatorname{ch}(t/2) + 3]},$$

$$n = A\int_0^R \operatorname{sh}^3\frac{t}{4}\, r^2\, dr, \quad (2.4)$$

$$M = u(R)\left[1 - \frac{u(R)}{2R}\right] - \frac{1}{2}\int_0^R \frac{u^2(r)\, dr}{r^2}.$$

Most Important Parameters of Configurations Consisting of a Degenerate Ideal Neutron Gas

	According to Einstein's theory				According to Newton's theory;				
$t(0)$	R	M	$n \cdot 10^{-56}$	$\Delta M/M_0$	R	$u(R)$	M	$n \cdot 10^{-56}$	$\Delta M/M_0$
1	1.5133	0.0324595	3.60178	0.008249	1.55668	0.0354367	0.0347413	3.86171	0.009977
2	0.9583	0.0658900	7.44366	0.025880	1.05537	0.0901267	0.0833362	9.53554	0.038240
3	0.6696	0.0766367	8.74018	0.035075	0.805802	0.1391840	0.1171330	14.0320	0.081375
4	0.5074	0.0710653	8.02574	0.025570	0.640732	0.1694480	0.1259460	16.0083	0.134190
5	0.4060	0.0598619	6.56506	−0.003436	0.521618	0.1778630	0.1142420	15.5392	0.190940
6	0.3640	0.0492384	5.18517	−0.045005	0.434748	0.1689490	0.0928140	13.5404	0.245670
7	0.3670	0.0419996	4.26480	−0.083740	0.374142	0.1503140	0.0709330	11.0402	0.292940
8	0.4130	0.0396985	3.97905	−0.097925	0.336988	0.1290290	0.0535240	8.75917	0.327540
10	0.4810	0.0458412	4.72655	−0.067306	0.333320	0.0970354	0.0362572	6.09621	0.345490
11	—	—	—	—	0.365932	0.0918297	0.0371440	5.92353	0.309930
12	0.4530	0.0473537	4.91175	−0.06095	0.404375	0.0948718	0.0424497	6.46310	0.277200
∞	0.4506	0.0458743	4.72432	−0.06858	0.362325	0.1092030	0.0435033	7.43305	0.355920

Notes on table: $t(0) = 4 \operatorname{arsh}\left[(3\pi^2)^{1/3} hN(0)^{1/3}/m_n c\right]$, where N(0) is the neutron density at the center, R the star radius, M the observed mass, u(R) the star mass neglecting Newtonian attraction (plane universe), n the number of neutrons in the star, and $\Delta M/M_0$ the gravitational packing factor ($\Delta M = M_0 - M$, $M_0 = nm_n$). Dimensions are given in units $c = k = 1$, $K_n = 1/4\pi$. To express mass in units of solar mass and radius in kilometers the tabular data should be multiplied by 9.29 and 13.7, respectively.

Here the first equation defines the star mass (we recall that the mass density $\rho = (\text{sh}\, t - t)/4\pi$, without taking into account the gravitational interaction between stars). For ordinary stars u(R) coincides with great accuracy with the observed mass. In the case of neutron configurations, where the mass defect is comparable to the mass, the value of u(R) differs appreciably from the latter. The second equation was derived from the condition of equality between the pressure gradient and Newtonian attraction. In the last relation M is the true star mass. It is defined as follows:

$$M = 4\pi \int_0^R \left[1 + \frac{1}{2}\varphi(r)\right] \rho(r)\, r^2\, dr, \tag{2.5}$$

where ρ is the mass density, and $\varphi(r)$ is the gravity potential at distance r (we recall that c = k = 1). It is easy to show that

$$\varphi(r) = -\frac{u(r)}{r} - 4\pi \int_r^R \rho(r)\, r\, dr. \tag{2.6}$$

After substituting (2. 6) into (2. 5) and making a number of simple transformations, we arrive at the last relation of system (2. 4). As before, the initial data for (2. 4) are determined by conditions (2. 3).

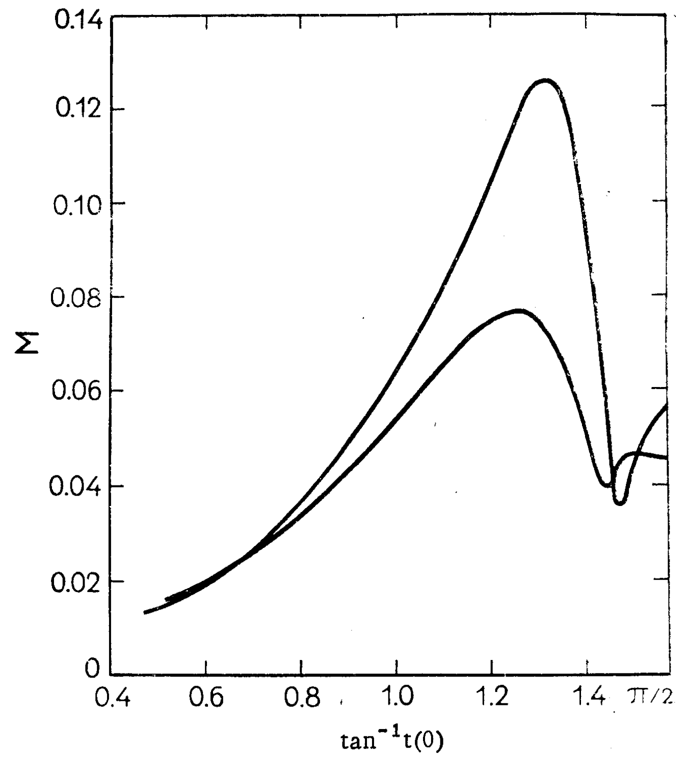


Fig. 1. Dependence of the mass of neutron configurations on the parameter $\tan^{-1} t(0)$ according to Einstein's and Newton's gravitational theories (lower and upper curves, respectively). The parameter t(0) is related to the central density by relation (2. 2).

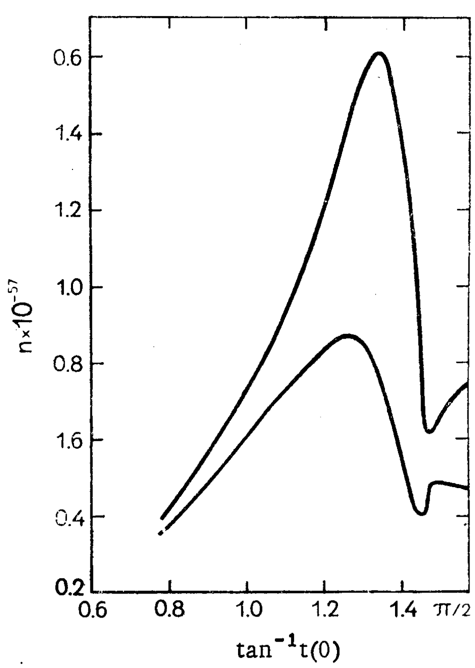
Fig. 2. (Right) Dependence of the number of neutrons in neutron stars on the parameter $\tan^{-1} t(0)$ according to Einstein's and Newton's gravitational theories (lower and upper curves, respectively).

The results of numerical computations of the most important parameters of the neutron configurations are given in the table. We are particularly interested here in the gravitational packing factor

$$\frac{\Delta M'}{M_0} = \frac{M_0 - M}{M_0}, \qquad M_0 = nm_n. \tag{2.7}$$

The values of the physical quantities having dimensionality are given in the table in units c = k = 1, $K_n = 1/4\pi$.

3. The tabulated data cannot give a graphic idea of the dependence of the characteristics of the configurations on the number of baryons that they contain or on the density at the center. Therefore we also present curves of some of the most important parameters. Figures 1 and 2 show the dependence between the mass of the configurations and the number of baryons that they contain on values of the parameter $\tan^{-1}t(0)$, where t(0) is determined by the neutron density at the center N(0) according to formula (2. 2). Note the surprising similarity of the curves for the mass and number of neutrons. Figures 3 and 4 show curves of the packing factor $\Delta M/M_0$. The captions for the



figures give a sufficient idea of the significance of the curves and therefore no detailed discussion of them is required. We shall note only a few common and important aspects of the relations represented in the figures.

a) Although the results of computations of the parameters of neutron stars on the basis of Newton's and Einstein's gravitational theories reveal an appreciable quantitative difference, nevertheless, in some most important respects there is good qualitative agreement. For example, Newton's theory of the mass, radius, and number of baryons in a star gives the correct orders of magnitude. Moreover, the curves for these parameters [e. g., for M(x) and n(x), where $x \equiv \tan^{-1}t(0)$] are quite similar.

b) According to both theories, all the parameters of the configurations are single-valued functions of the central density N(0) [or, what is equivalent, the parameter t(0)]. The opposite assertion is, in general, untrue. In certain regions the same value of some of the star parameters, such as mass, correspond to two (or even more) values of the central density (this is discussed in [1, 3, 4, 10]).

c) According to Einstein's gravitational theory, the curves of the dependence of the star parameters on x have a number of maxima and minima. Figures 1, 2, and 3 clearly show two maxima and one minimum for t(0) = 3.34, 12.35, and 8.24 ($N(0) = 3.0 \cdot 10^{39}$, $4.84 \cdot 10^{42}$, and $2.1 \cdot 10^{41}\,cm^{-3}$). However, beyond the latter maximum at x = 1.49 there are many extrema not shown in the figures. When x > 1.5 the curves, oscillating (with a strongly damped amplitude), tend to a definite limit as the density at the center tends to infinity. These oscillations of the curves have been investigated in [5] for the case of relativistic densities.

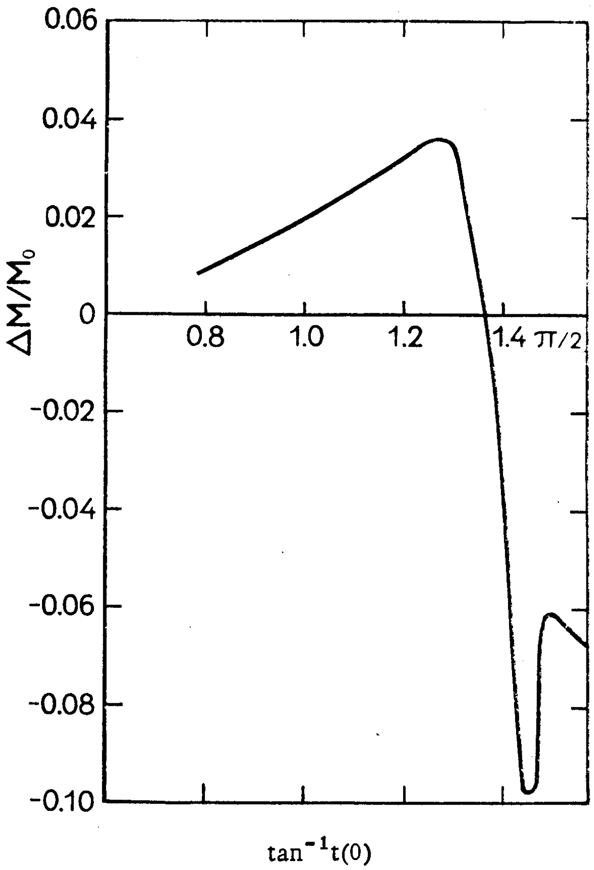


Fig. 3. Dependence of the packing factor for neutron configurations on the parameter $\tan^{-1}t(0)$ according to Einstein's gravitational theory. The packing factor is defined in (2.7).

d) In the case of Newtonian models, the absolute gravitational mass defect for all possible static configurations has a positive value. As can be seen from the last column of the table, the packing coefficient is in all cases an increasing function of the central density, with the exception of the region of extraordinarily high central densities (see the next to last line of the table for $t(0) \approx 12$), where a conspicuous minimum is recorded.

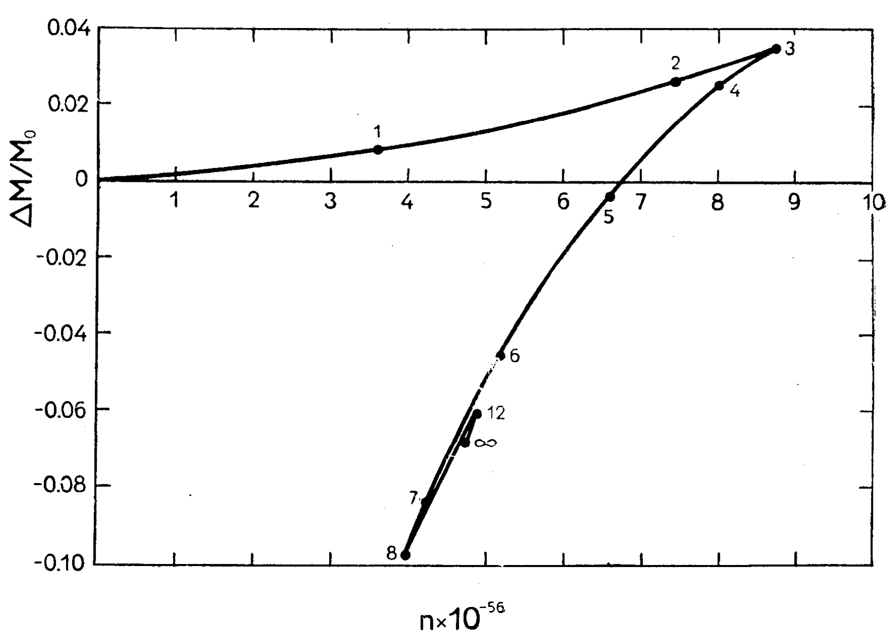


Fig. 4. Dependence of packing factor for neutron configurations on the total number of neutrons they contain. The figures on the curve indicate the corresponding values of the parameter t(0) for the points denoted by black dots.

In the case of relativistic models, the gravitational packing factor exhibits anomalous behavior (see Fig. 3). At first, it increases with increase in density, at x = 1.27 (t(0) = 3.34, N(0) = 3 · 10^{39}cm^{-3}) it attains a maximum and then begins to decrease.

When x > 1.36 (t(0) > 4.67, N(0) > 1.12 · 10^{40}cm^{-3}) the packing factor becomes negative. At the point x = 1.45 (t(0) = 8.2, N(0) = 2 · 10^{41} cm^{-3}) there is a deep minimum, approximately equal to −0.1; then the packing factor, oscillating with a small and strongly damping amplitude and continuing to remain negative as $\rho(0) \to \infty$, tends to a limit of −0.069.

e) The curve of the dependence of the packing factor on the number of neutrons in a star $\Delta M/M_0 \equiv f(n)$ has an interesting shape (Fig. 4). The figures associated with the black dots on the curve denote the corresponding values of the parameter t(0). Note that at the points where t(0) = 3, 8, 12, etc. the slope has discontinuities. The existence of these discontinuities becomes obvious when we note that before and after these points the derivative $f'(n)$ has the same (in this case positive) sign. This behavior of the M(n) curve was commented on in [2]. Taking into account the fact that [2]:

$$\frac{dM}{dn} = m_n\left(1 - \frac{2M}{R}\right)^{1/2},$$

for the derivative of the function $\Delta M/M_0$ we find

$$\frac{d}{dn}\left(\frac{\Delta M}{M_0}\right) = \frac{Mm_n}{M_0^2}\left[1 - \frac{M_0}{M}\left(1 - \frac{2M}{R}\right)^{1/2}\right]. \tag{3.1}$$

This expression cannot become equal to infinity; moreover, by using the data given in the table we can confirm that in fact it always has a positive sign. Beyond the point t(0) = 12 the function f(n) also oscillates, but due to the close spacing of the zigzags and the rapid decay of amplitude it is difficult to trace this oscillation.

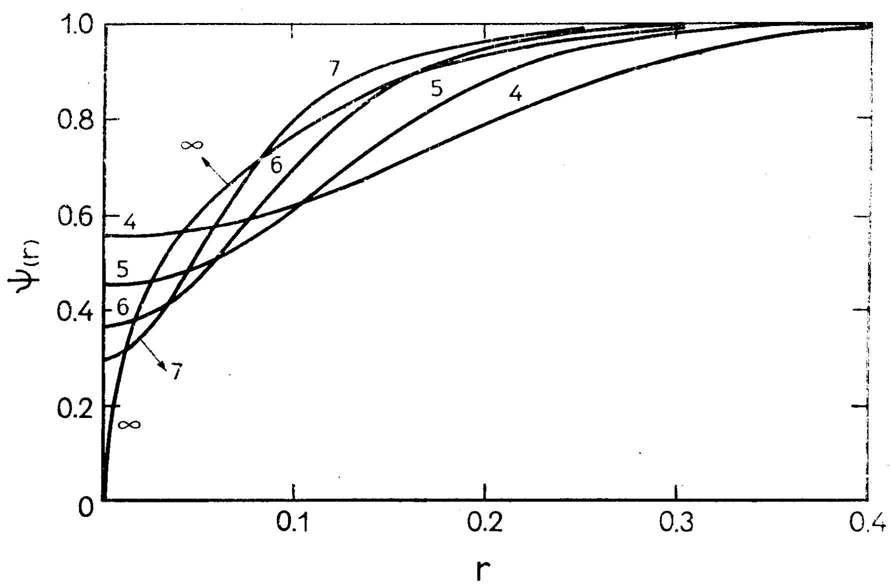


Fig. 5. Dependence of the function $\Psi(r) = \sqrt{-g_{00}(r)\, g_{(rr)}(r)}$ on radial distance; g_{00}(r) and $g_{(rr)}$(r) are the temporal and radial components of the metric tensor. The distances are measured in units c = k = 1 and $K_n = 1/4\pi$. The values of the function Ψ(r) give some idea of the degree of deviation from additivity of the internal energy of the star.

The anomalous behavior of the mass defect will now be discussed in greater detail. We have seen that with respect to mass defect Newton's and Einstein's gravitational theories give results which are both qualitatively and quantitatively different. This comparison obviously indicates that here an important role is played by the curvature of space, that is, an effect of the general theory of relativity is involved.

It follows from the condition of equilibrium along the radius of a star that

$$E_n(r)\sqrt{-g_{00}(r)} = m_n(1 - 2M/R)^{1/2} < m_n, \tag{3.2}$$

where E_n(r) is the limiting energy of neutrons at distance r, g_{00} is the temporal component of the metric tensor and M and R are the star mass and radius, respectively. Note that, in contrast to heated stars, in superdense cold bodies a sharp boundary exists. Thus, in the case of neutron configurations the radius is determined from the condition $E_n(r) = m_n$, that is, the surface of the body is located where the Fermi limit intersects the wall of the "gravitational potential well." It can be seen from (3. 2) that the highest Fermi level lies in the potential well. This means that the individual particles in the star form a bound state, and therefore they cannot escape it. On the other hand, computations show that for some of the densest configurations

$$M > \sum_k n_k m_k . \tag{3.3}$$

How can these two facts be reconciled? Actually, inequalities (3. 2) and (3. 3) are not contradictory if we consider that the total internal energy of the star is not equal to the sum of the energies (kinetic, potential) of the particles of which it consists.

If the internal energy possessed the property of additivity, then for the mass we would have

$$M_1 = 4\pi \int_0^R \Psi(r)\rho(r) r^2 dr < nm_n , \tag{3.4}$$

where n is the number of neutrons in the star and

$$\Psi(r) = \sqrt{-g_{00}(r)\, g_{rr}(r)}. \tag{3.5}$$

On the other hand, according to the first equation of system (2. 1), the observed mass of the star is

$$M = 4\pi \int_0^R \rho(r) r^2 dr. \tag{3.6}$$

A comparison of (3. 4) and (3. 6) shows that the energy does not possess the property of additivity. The range of variation of Ψ will serve as a measure of its deviation. Fig. 5 shows curves of this function for configurations with t(0) = 4, 5, 6, 7, and ∞. They are similar to $-g_{00}(r)$ curves (see Fig. 4 in [1]). Ψ has a minimum at the center; with increase in r it increases monotonically and at the surface differs little from unity. In the case of rigorous adherence to additivity we would have $\Psi = 1$, and the maximum deviation will occur when $\Psi = 0$. As the parameter t(0) increases, $\Psi(r)$ in the central region decreases. Thus, the greatest deviation from additivity is associated with the central part.

An idea of the region of the star with which negative mass defect is associated is most graphically conveyed by Fig. 6, which shows curves of the function

$$\Phi(r) = \left[1 - \left(1 - \frac{2u(r)}{r}\right)^{1/2} \frac{\rho(r)}{m_n N(r)}\right]\left(1 - \frac{2u(r)}{r}\right)^{-1/2} N(r)\, r^2, \tag{3.7}$$

where N(r) is neutron density. The integral of this function determines the mass defect

$$\Delta M = 4\pi m_n \int_0^R \Phi(r)\, dr. \tag{3.8}$$

For small r the function $\Phi(r)$ is negative; it has one minimum and one maximum, and vanishes at the center and at the surface. With increase in t(0) there is an increase in the role of the area situated below the x-axis, and when t(0) > 4. 7 the algebraic sum of the areas enclosed by the $\Phi(r)$ curve and the x-axis becomes negative.

We therefore conclude that the mass defect anomaly was caused by a catastrophic deviation from additivity of the internal energy due to warping of the spatial metric in the corresponding baryon configurations. In these configurations and in the Newtonian approximation the deviation from additivity (here the kinetic energy is additive and the potential energy is not) is strong, but inadequate for a change in the sign of the mass defect. In fact, from the virial theorem it follows [13] that

$$-\Delta M = \sum_k m_k c^2 \left(\frac{m_k c^2}{E_k} - 1\right) > 0. \tag{3.9}$$

Note that for proof of (3. 9) the assumption of periodicity or quasi-periodicity of motion of the particles is not mandatory.

4. We now turn again to Fig. 4. Obviously, the configurations corresponding to the lower branch of the $\Delta M/M_0 = f(n)$ curve when 3 < t(0) < 4. 67 are unstable in relation to the transitions to the upper branch, where the mass defect exhibits normal behavior. However, configurations with t(0) > 4. 67, having a negative absolute mass defect, are unstable not only in relation to transition to the upper branch, but also in relation to decay into a diffuse state. Since the

mass defect is several percent of the mass of the star itself, an improbably large energy will be released in these transitions. The energy associated with one gram of star matter is an order of magnitude greater than the corresponding energy released in thermonuclear reactions in the combustion of hydrogen. It is important to note that the binding energy of each particle in a star is negative, so that the particles cannot escape individually to infinity. The escape of a certain number of baryons from a star requires the addition of supplementary energy to the remaining configuration from the outside. For this reason it cannot occur spontaneously. This means that transition of the system to a more stable state can occur only under the influence of very great perturbations. In this case expansion will occur, accompanied by heating of the celestial body. The corresponding transition will have the character of a cosmic explosion. These arguments concerning the fate of baryon configurations with an anomalous absolute mass defect were first presented in [1].

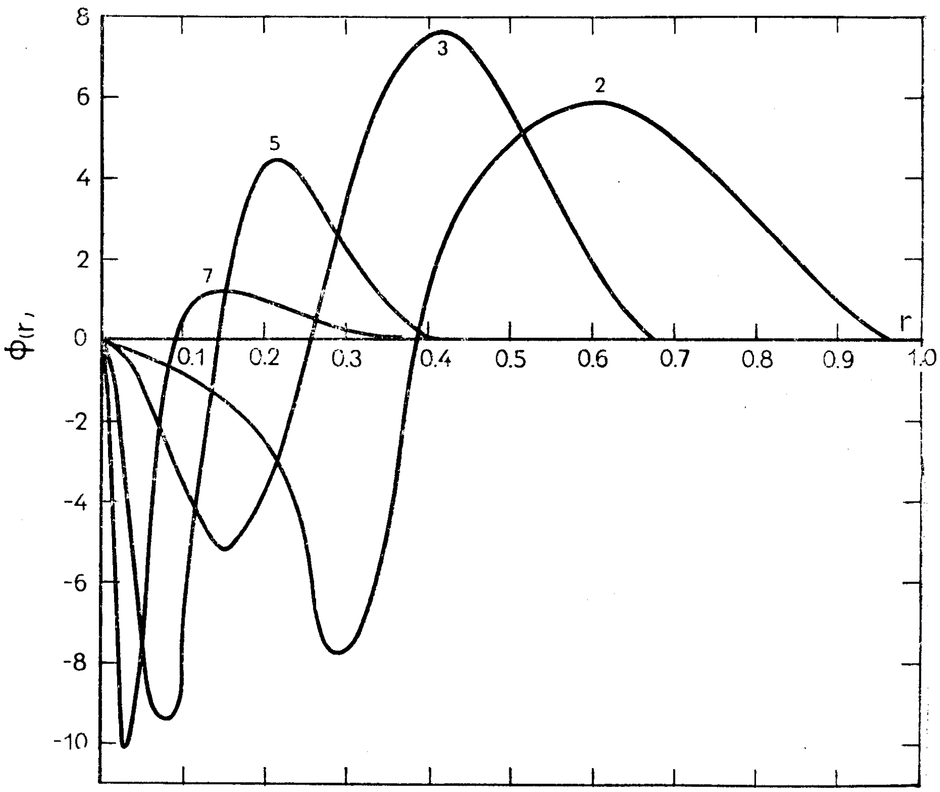


Fig. 6. The distance from the center of the star is plotted along the x-axis in the units used in this paper. Values of the function $\Phi(r)$ (Einstein's theory) are plotted along the y-axis. The figures on the curves indicate the values of the parameter $t(0)$ for corresponding configurations. Along the y-axis the scale differs: for $t(0) = 2$, 3, 5, and 7 the units used are a, $2a$, $5a$, and $10a$, respectively, where a is a known constant number.

We feel that configurations with an anomalous value of the absolute mass defect (we refer to the entire branch of the curve with $t(0) > 3$) are of some importance to cosmogony.

According to one modern cosmogonic concept [8, 9], stars, different star groups, and the interstellar gas are formed from certain superdense prestellar bodies by the eruption of various quantities of matter.

In order of magnitude the mass of the prestellar body must be greater than the mass of a star, whereas above we discussed static celestial bodies with a mass of the order of the solar mass or even less. In order to relate the above considerations on the behavior of baryon configurations with an anomalous value of the absolute mass defect to the concept mentioned, it is necessary to construct models of superdense prestellar bodies with masses of a much greater order of magnitude than the solar mass. This would solve in principle the problem of "superdense" cosmogony. However, the construction of physical models of continuous superdense prestellar bodies of great mass involves difficulties.

The solution of these difficulties may possibly involve the consideration of nonstationary and nonequilibrium models. A recent study by I. D. Novikov [12] is of interest in this connection; Novikov postulates that at some initial time, when the density was infinitely great, not all the matter expanded uniformly, certain regions lagging in their development. These regions can be identified with prestellar bodies.

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